# Statistics 201 Exam 3 – Practice Exam Fall 2025 Confidence Intervals – Chapter 19 With KEY

#### Disclaimer:

This practice exam is provided solely for the purpose of familiarizing you with the format and style of the Stat 201 exams. There is no explicit or implicit guarantee that the upcoming exam will ask similar questions. If you use the practice exam as your only tool to help you prepare for the upcoming exam, you most likely will not do well on the exam. You should still do the things you would have done if you did not have access to this practice exam, such as re-read the text, go over your class notes, and look at the list of exam topics provided and make sure that you understand all the concepts listed within it.

NOTE: These questions come from actual older exams. As such, the points on this practice exam may total more than 100 points.



**Please note:** when asked to write out null and alternative hypotheses, if appropriate, be sure to use proper mathematical notation (i.e., use symbols such as  $\langle =, \leq, \geq, \neq, \rangle, p, \hat{p}, \mu, \bar{x}, \text{ etc.}$ ).

Exam Grade: <u>100</u>

## ON THIS EXAM, WHEN ROUNDING A FINAL NUMERICAL ANSWER, PLEASE REPORT AT LEAST 3 SIGNIFICANT DIGITS.

EXAMPLE: the value 0.000378516 should be rounded to 0.000379

1. Below is output from a hypothesis test for a single population mean based on 28 observations. The null hypothesized value, as well as the sample mean ("actual estimate") are seen below (among other numbers). Calculate the 3 missing values below. Show your work (where necessary), but fill in your final answers in the blanklines below.

Summary Statistics				
y-bar	18.6286			
S	2.46484			
sample size	28			
Hypothesized Value	20			
Standard Error	0.4658			
DF				
Test Statistic				
P-value >  t				
P-value > t	0.9967			
P-value < t	0.0033			

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i) (2 points) DF: 
$$df = n - 1 = 28 - 1 = 27$$

ii) (2 points) Test Statistic: 
$$\frac{\overline{y} - \mu_0}{SD(\overline{y})} = \frac{18.6286 - 20}{\frac{2.46484}{\sqrt{28}}} = \frac{-1.3714}{0.46581} = -2.944$$

iii) (2 points) Prob > |t|: 2(0.0033) = 0.0066

- 2. It is generally believed that 75% of college sophomores regularly recycle. Are UT sophomores different from this widely accepted figure? A researcher at UT got the cooperation of the registrar's office and was given the contact information for a large number of randomly selected sophomores at UT (out of the approximately 5,000 UT sophomores). The researcher got responses to a short survey from 128 of these students. Based on the answers they provided, the researcher determined that 84 of the 128 students regularly recycled.
  - i) (2 points) Suppose the researcher wishes to carry out a formal hypothesis test for their research question. Write out the null and alternative hypotheses they wish to test.

$$H_0$$
:  $p = 0.75$   
 $H_A$ :  $p \neq 0.75$ 

ii) (6 points) Are the 3 conditions for performing a hypothesis test for a proportion met in this case? State each condition, indicate whether you think each condition is met, and explain your reasoning. Provide numerical justification whereappropriate.

Condition 1 is: Randomization

Is condition 1 met? (Circle one): Briefly explain your answer:



The Problem states a random sample of UT sophomores was selected.

Condition 2 is: 10% Condition

Is condition 2 met? (Circle one): Briefly explain your answer:



128 is less than 10% of the approx. 5000 sophomores.

Condition 3 is: **Success/Failure Condition** 

Is condition 3 met? (Circle one):



Briefly explain your answer:

are both  $n \times p_0$  and  $n \times q_0 \ge 10$ ?  $n \times p_0 = 128(0.75) = 96$ 

 $n \times q_0 = 128(0.25) = 32$ 

Both are above 10, so the condition is met

#### Question 2 (continued)

iii) (3 points) Calculate the test statistic for the hypothesis test.

$$z = \frac{\widehat{p} - p_0}{\sqrt{\frac{p_0 \times q_0}{n}}}$$

$$\widehat{p} = \frac{84}{128} = 0.65625$$

$$z = \frac{0.65625 - 0.75}{\sqrt{\frac{0.75 \times 0.25}{128}}}$$

$$z = \frac{-0.09375}{0.03827} = -2.450$$

iv) (2 points) Using an alpha level of 0.07, with the resulting p-value of 0.0143 what is your conclusion regarding your null hypothesis in part (i)? Circle the best answer:

Reject the null hypothesis

Fail to reject the null hypothesis

v) (4 points) Write out the *full* conclusion of the hypothesis test. Be sure tostate your conclusion in the context of this problem.

Reject the null hypothesis, with a level of significance of 0.07 and a p-value of 0.0143, there is sufficient evidence to conclude that the proportion of UT Sophomores who recycle is different than 75%.

vi) (2 points) What would be the next step for the research after the finding the results hypothesis test?

Since the null hypothesis was rejected and now the population proportion is now considered unknown, the research could:

Create a Confidence Interval using the sample data where they believe the true proportion of UTK Sophomores who recycle

OR

Conduct another Hypothesis Test, using their p-hat as the new hypothesized value

- 3. A researcher believes that British men are shorter, on average, than men from the USA. A random sample of 35 men from the USA (ages 20-50) was observed for height. The mean height was 68.1 inches and the sample standard deviation was 5.34 inches. Meanwhile, a random sample of 35 men from Britain (ages 20-50) was observed for height. The mean height was 64.6 and the sample standard deviation was 5.81 inches.
  - i) (2 points) Which type of problem is this? Circle the best answer:
    - a) One sample proportion test
    - b) One sample test of a mean
    - Two independent samples t-test
    - d) Chi-Squared test of independence
  - ii) (2 points) Write down the null and alternative hypotheses suggested by the first sentence of this problem (HINT: to avoid confusion later, look at the top line of the JMP output below <u>now</u>).

$$\begin{aligned} &H_0: \mu_{USA} - \mu_{Britian} = 0 \\ &H_A: \mu_{USA} - \mu_{Britian} > 0 \end{aligned}$$

iii) (3 points) Below is some output from an analysis of these data. Calculate the missing Test Statistic.

$$t = \frac{3.4971}{1.33411} = 2.621$$
Longer Way:
$$t = \frac{68.1 - 64.6}{\sqrt{\frac{(5.34)^2}{35} + \frac{(5.81)^2}{35}}} = \frac{3.5}{1.3337} = 2.624$$

Summary Statistics				
Sample Difference	3.50			
Standard Error	1.33			
DF	67.52			
Test Statistic				
P-value >  t	0.0108			
P-value > t	0.0054			
P-value < t	0.9946			

iv) (1 point) There are 3 p-values displayed in the output. Circle the p-value that corresponds to your alternative hypothesis in part [ii] above.

Question 3 (continued)

v) (1 point) If we use an  $\alpha$  level of 0.01, what is your conclusion regarding your null hypothesis in part (ii)? Circle the best answer:

Reject the null hypothesis

Fail to reject the null hypothesis

vi) (3 points) State the remainder of your conclusion for this hypothesis test. Be sure to state your conclusion in the context of this problem.

There is sufficient evidence to conclude, at the 0.01 level of significance, that men ages 20-50 from USA are taller, on average, than men ages 20-50 from Great Britain.

4. (2 points) The following is some output from JMP for a one-sample test of a population mean. The 95% confidence interval for the mean is showing, but much of the other output has been erased. The researcher wanted to do a two-sided hypothesis test, and was testing to see if the population mean is 22 (as indicated in the output).

Confidence Interval

**Hypothesis Test** 

Summary Statistics		
y-bar	24.03	
S	3.477	
sample size	33	
Confidence Level	95%	
Standard Error	0.6053	
DF	32	
Critical Value (t*)	2.0369	
Margin of Error	1.2329	
Lower Limit	22.797	
Upper Limit	25.263	

Summary Statistics		
y-bar	24.03	
s	3.477	
sample size	33	
Hypothesized Value	22	
Standard Error	0.6053	
DF		
Test Statistic		
P-value >  t		
P-value > t		
P-value < t		

Based on the output that is still showing, what is a possible value for the p-value next to "Prob > |t|"? Circle the best answer:

- a) 1.0
- b) 0.9999
- c) 0.5015
- d) 0.0501
- **(e)** 0.0051
- f) The output that is still showing offers no clue as to what that p-value might be.

(Since 22 is <u>not</u> inside the 95% confidence interval, Prob >|t| <u>has</u> to be less than 0.05)

- 5. The American Banking Association is concerned that the percentage of customers who default on their auto loans is too high. They are basing this belief on a 95% confidence interval of the population proportion constructed from a random sample of loans. The confidence interval was (0.1631, 0.2329).
  - i) (2 points) What is the margin of error?

$$\frac{(0.2329-0.1631)}{2}=0.0349$$

ii) (2 points) What is the sample proportion?

Either
$$UL - MOE: 0.2329 - 0.0349 = 0.198$$

$$OR$$

$$LL + MOE: 0.1631 + 0.0349 = 0.198$$

iii) (4 points) Interpret this interval in the context of the problem.

I am 95% confident the true proportion of people who default on their auto loans is contained in the interval 0.1631 to 0.2329.

Note: describing the interval using percentages (i.e.16.31% to 23.29%) would be acceptable.

iv)(2 points) Given that the critical value Z\*=1.96, what is the value of the standard error used in constructing the confidence interval?

Margin of Error = Critical Value x Standard Error 
$$0.0349 = 1.96 \times SE$$
  
 $SE = 0.0178$ 

6.	Suppose that the Business Analytics and Statistics Department has come up with a method to determine whether or not a student cheated on their exam. We test these simple hypotheses as follows:
	H <sub>0</sub> : The student didn't cheat H <sub>A</sub> : The student cheated
	<ul> <li>i) (2 points) With any test there is always the risk of making the wrong decision.</li> <li>Describe what a type I error is in the context of this problem.</li> </ul>
	(Definition: $H_0$ is true, but we rejected it)
	The student, in reality, didn't cheat, but we conclude that they did.
	ii) (2 points) Describe what a type II error is in the context of this problem.
	( $H_0$ is false, but we didn't reject it)
	The student, in reality, did cheat, but we conclude that they did not.
	iii) (2 points) If we were to give a 0 to any student who we determined to have cheated using the Business Analytics and Statistics Department's detection method, which type of error would a student typically consider more serious? Briefly explain why.
	Type I. Being an honest student that is accused of cheating is much worse from a student's perspective.
	iv)(2 points) What is the symbol used to identify the probability of making a type II error? (Circle One):
	$\alpha$ (1- $\alpha$ ) $\beta$ (1- $\beta$ ) $\chi^2$

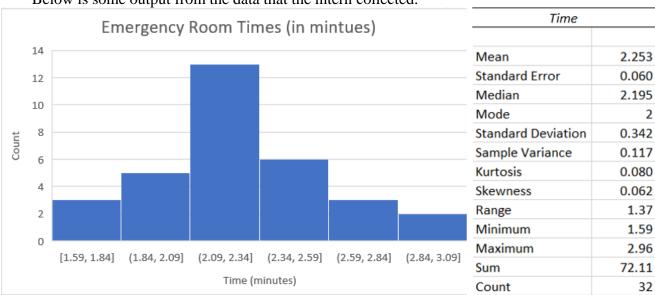
7. As patients arrive at a local hospital emergency room, as long as they are able to respond, and are not deemed to have an immediate life threatening condition, they are asked some standard questions before being asked to take a seat in the waiting room. The time it takes to complete this initial set of questions has come under scrutiny lately. The emergency room manager believes that the average amount of time it takes to get answers to these initial questions is 2 minutes. Most emergency room employees believe it takes longer than that, on average.

A nursing school intern was asked to randomly select incoming patients over a 2 week period, and time how long it takes to get answers to these initial questions. The following are some of the results of the intern's data collection efforts.

i) (2 points) Write out the null and alternative hypotheses suggested by the introduction to this problem.

$$H_0$$
:  $\mu = 2min$   
 $H_A$ :  $\mu > 2min$ 

Below is some output from the data that the intern collected:



Question 6 (continued)

ii) (3 points) Use the information in the output to calculate the value of the test statistic for this hypothesis test.

$$t = \frac{\overline{y} - \mu_0}{SD(\overline{y})} = \frac{2.253 - 2}{\frac{0.342}{\sqrt{32}}} = \frac{0.253}{0.0605} = 4.18$$

iii) (2 points) Based on your answer to part [ii] above, what would your decision be regarding the null hypothesis? Use  $\alpha = 0.05$ . Circle the best answer:

Reject the null hypothesis

Fail to reject the null hypothesis

Briefly explain your reasoning for your selection above.

(NOTE: If you have NO ANSWER to part [ii], use (3.21) as your answer to part [ii], and use that to answer part [iii]).

The t-distribution is a lot like the standard normal distribution. Being beyond 3 standard errors from the hypothesized values will lead to a very small p-value

iv) (3 points) State the remainder of your conclusion for this hypothesis test. Be sure to state your conclusion in the context of this problem.

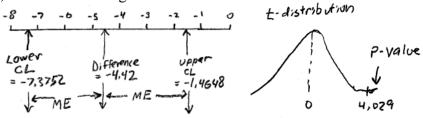
There is sufficient evidence to conclude, at the 0.05 level of significance, that the average amount of time it takes to get answers to these initial questions is greater than 2 minutes.

8. Below is some output for a two-sample confidence interval. Answer the following questions based on the output provided.

	Group 1	Group 2
Group Name	Group A	Group B

Summary Statistics		
Sample Difference	-4.42	
Standard Error	1.47	
DF		
Confidence Level	95%	
Lower Limit	-7.3752	
Upper Limit	-1.4648	

(2 points) What is the "margin of error" of this confidence interval?



Can use Upper Limit:

$$ME = -1.4648 - (-4.42) = 2.955$$

**OR Lower Limit** 

$$ME = -4.42 - (-7.3752) = 2.955$$

OR use Both Upper and Lower Limits 
$$ME = \frac{-1.4648 - (-7.3752)}{2} = 2.955$$

ii) (2 points) What was the value of  $t_{dif}^*$  that was used to construct this confidence interval? Please report 4 decimal places.

$$(\overline{y}_1 - \overline{y}_2) \pm t_{df}^* \times SE(\overline{y}_1 - \overline{y}_2) -4.42 \pm t_{df}^* \times 1.4713$$

**Solve using Lower Limit:** 

$$-7.3752 = -4.42 - t_{df}^* \times 1.4713$$
  
 $-2.9552 = -t_{df}^* \times 1.4713$   
 $t_{df}^* = 2.0086$ 

Or Solve using Upper Limit:

$$-1.4648 = -4.42 + t_{df}^* \times 1.4713$$
  
 $2.9552 = t_{df}^* \times 1.4713$   
 $t_{df}^* = 2.0086$ 

9. The Gallup Poll conducted a representative telephone survey during the first quarter of 2010 in the United States. Among the reported results was the following table concerning the preferred political party affiliation of respondents and their ages. We are interested in seeing if there is an association between a person's age and their political party affiliation in the United States.

Table of Political Party and Age Group

, , , , , , , , , , , , , , , , , , , ,				
Count	Democrat	Independent	Republican	Total
18-29	131	209	51	391
30-49	140	200	79	419
50-64	141	145	102	388
65+	162	113	129	404
Total	574	667	361	1602

i) (2 points) Write the appropriate null and alternative hypothesis suggested by the statements above.

 $H_0$ : Political Party affiliation is independent of which age group a person is in

 $H_A$ : Political Party affiliation is <u>not</u> independent of which age group a person is in

ii) (4 points) Below is some output for this analysis. Two of the values in the table have been erased (A and B). Provide the necessary arithmetic for these two missing values.
 Write your final answers in the blanks: A 162.795
 B 2.0547

Expected	Democrat	Independent	Republican
18-29	140.096	Α	88.109
30-49	150.129	174.453	94.419
50-64	139.021	161.546	87.433
65+	144.754	168.207	91.039

9
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A) Expected Cell Count =
$$\frac{3}{9}$$
 $\frac{391 \times 667}{1602} = 162.795$ 

Cell χ2	Democrat	Independent	Republican
18-29	0.5906	13.1143	15.6294
30-49	0.6833	3.7413	2.5179
50-64	0.0282	1.6946	2.4269
65+	В	18.1195	15.8291

B) Cell Chi – Square = 
$$\chi^2$$
 =  $\frac{(162-144.754)^2}{144.754}$  = 2.0547

Question 8 (continued)

iii) (3 points) As with any statistical technique, there are conditions that must be met for the analysis to be valid. One of the conditions of this technique (the Chi-Square test of independence) has something to do with there being enough data for the analysis to be valid. What is the name of this condition, and do you believe it is met in this case? Provide a brief explanation for your "yes" or "no" answer.

Expected Cell Count condition. Yes, it is met. The smallest expected cell count is 87.4332. Since all expected cell counts are at least 5, the condition is met.

iv) (2 points) From the data given, calculate the degrees of freedom for the chi-square test of independence.

$$df = (\#Rows - 1) \times (\#Columns - 1) = (4 - 1) \times (3 - 1) = 6$$

v) (2 points) Below is more output for this analysis. The Test Statistic §76.43. Referring to the numbers in the Contingency Table at the bottom of the previous page, briefly explain how this number was calculated (use the space to the right of the output for your answer).

Summary Statistics		
Test statistic 76.43		
DF	3	
p-value	1.79E-16	

All of the "Cell Chi^2" values from the Contingency Table were added together for the overall Ch-Square test statistic

vi) (1 point) Using  $\alpha$ =0.005, what is your conclusion regarding your null hypothesis from part [i]? Circle the best answer:

Reject the null hypothesis

Fail to reject the null hypothesis

Briefly explain your reasoning for your selection above.

Since the p-value of <0.0001 is less than 0.005 (less than alpha), we reject  $H_0$ 

vii) (2 points) State the remainder of your conclusion for this hypothesis test. Be sure to state your conclusion in the context of this problem.

There is evidence to suggest at the 0.005 level of significance, that political party affiliation is related to a person's age in the USA (at least when this survey was taken!).

#### Question 8 (continued)

Sum of Freq

viii) (4 points) The following is the stacked bar chart for this analysis. In one or two sentences, how would you describe the nature of the association that seems to exist between these two variables?

Stacked Bar Chart of Political Party by Age Group 100% 90% 80% 70% 60% 50% Politcal Party ▼ 40% 30% ■ Republican 20% ■ Independent 10% 0% Democrat 18-29 30-49 65+ 50-64 ■ Republican 79 51 102 129 ■ Independent 209 200 145 113 141 162 Democrat 131 140

As Americans get older, they seem to drift away from being "independents"/ These people seem to become both republicans and democrats, but slightly more often become republicans.

Age Group

- 10. (3 points) Does studying really pay off? Is there a relationship between the total number of minutes students spent preparing for all 3 exams in Stat 201 and the average score they achieved on these exams? What tool or technique would you use to address this question? Circle the best answer:
  - a) Box plot
  - b) Hypothesis test for a population mean
  - c) Decision tree
  - d) Confidence interval for the difference between two population means
  - Simple linear regression
- 11. (3 points) You are the regional manager of 5 large grocery stores. You let the manager of each of these 5 stores make their own decisions regarding who to hire. Since these 5 stores are within 50 miles of each other, you would expect the ethnic background of the employees at these 5 stores to be fairly similar. Is that the case? What tool or technique would you use to address this question? Circle the best answer:
  - a) Histogram
  - (b) Chi-Square test of independence
  - c) Simple linear regression
  - d) Hypothesis test for a population mean
  - e) Pearson's correlation coefficient (r)

True/False Questions Circle the best answer (2 points each)

- T (Power" represents the probability that we correctly fail to reject a true null hypothesis.
- F If, based on the same set of data, you change your level of confidence from 90% to 95%, the resulting 95% confidence interval is considered less precise than the 90% confidence interval.
- The A p-value represents the probability of seeing the results you saw, or results more unusual than that, assuming the null hypothesis is true.
- The only way to decrease the probability of a Type II error is to increase the probability of a Type I error.

### [YOU MAY REMOVE THIS SHEET]

#### **ONE SAMPLE TESTS**

#### **Proportion**

1 Topor don		
Confidence Intervals	$\widehat{p} \pm z^* \times SE(\widehat{p})$	$SE(\widehat{p}) = \sqrt{\frac{\widehat{p}\widehat{q}}{n}}$
<b>Hypothesis Testing</b>	$z_{\widehat{p}} = \frac{\widehat{p} - p_0}{\mathrm{SD}(\widehat{p})}$	$SD(\widehat{p}) = \sqrt{\frac{\overline{p_0q_0}}{n}}$

#### Mean

Confidence Intervals	$\overline{y} \pm t_{df}^* \times SE(\overline{y})$	$SE(\overline{y}) = \frac{s}{\sqrt{n}}$
<b>Hypothesis Testing</b>	$t_{df} = \frac{\overline{y} - \mu_0}{SE(\overline{y})}$	$SE(\overline{y}) = \frac{s}{\sqrt{n}}$

(*df*) degrees of freedom for the t-distribution = n - 1

#### TWO SAMPLE TESTS

#### **Difference Between Two Means**

Confidence Intervals	$\boxed{\overline{y}_1 - \overline{y}_2 \pm t_{df}^* \times SE(\overline{y}_1 - \overline{y}_2)}$	$SE(\overline{y}_1 - \overline{y}_2) = \sqrt{\frac{(s_1)^2}{n_1} + \frac{(s_2)^2}{n_2}}$
Hypothesis Testing	$t_{df} = \frac{(\overline{y}_1 - \overline{y}_2) - 0}{SE(\overline{y}_1 - \overline{y}_2)}$	$SE(\overline{y}_1 - \overline{y}_2) = \sqrt{\frac{(s_1)^2}{n_1} + \frac{(s_2)^2}{n_2}}$

(*df*) degrees of freedom for the t-distribution available from JMP output

#### **Test of Independence**

$$\chi_{df}^2 = Sum \ of \ \frac{(Obs-Exp)^2}{Exp} \quad \text{where} \quad Exp = \frac{Row \ Total \times Column \ Total}{N} \quad \text{for each cell.}$$

$$Degrees \ of \ freedom = (\#Rows - 1) \times (\#Columns - 1)$$